Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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12 pages

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

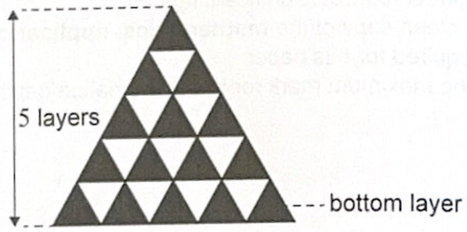
1. [Maximum mark: 16]

Thai cushions are designed with a triangular cross-section and are made from layers of smaller cushions. These cushions can be modelled as triangular prisms.

This is shown in the diagram.

diagram not to scale

Thai cushion with 4 layers



Cross-section of Thai cushion with 5 layers

(a) Write down the number of triangular prisms in the bottom layer of the cushion with

(i) 4 layers.

(ii) 5 layers. [2]

Mayumi notices that the number of triangular prisms in the bottom layer of the cushions forms an arithmetic sequence.

(b) (i) Write down the common difference of this sequence.

(ii) Find an expression for the number of triangular prisms in the bottom layer of a cushion with n layers. [3]

(This question continues on the following page)

(Question 1 continued)

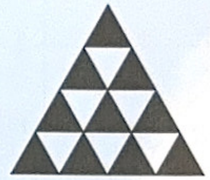
Mayumi wants to extend this design to create a cushion with 9 layers.

(c) (i) Find the number of triangular prisms in the bottom layer of Mayumi's cushion.

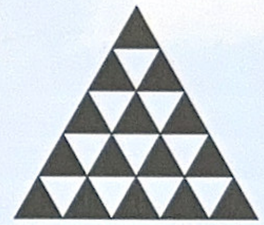
(ii) Calculate the **total** number of triangular prisms in Mayumi's cushion. [3]

(d) Find an expression for the **total** number of triangular prisms in a cushion with n layers, giving your answer in its simplest form. [2]

The cross-section of the cushion consists of black triangles and white triangles.



This cushion with 4 layers has a total of 6 white triangles.



This cushion with 5 layers has 4 white triangles in its bottom layer.

(e) Write down the total number of black triangles in a cushion with 4 layers. [1]

The number of black triangles in each layer forms an arithmetic sequence.

(f) Find and simplify an expression for the total number of black triangles in a cushion with n layers. [2]

The total number of white triangles in a cushion with n layers is $\frac{n(n-1)}{2}$.

(g) Using both the given expression and your answer to part (f), find and simplify an expression for the total number of black and white triangles in a cushion with n layers. [3]

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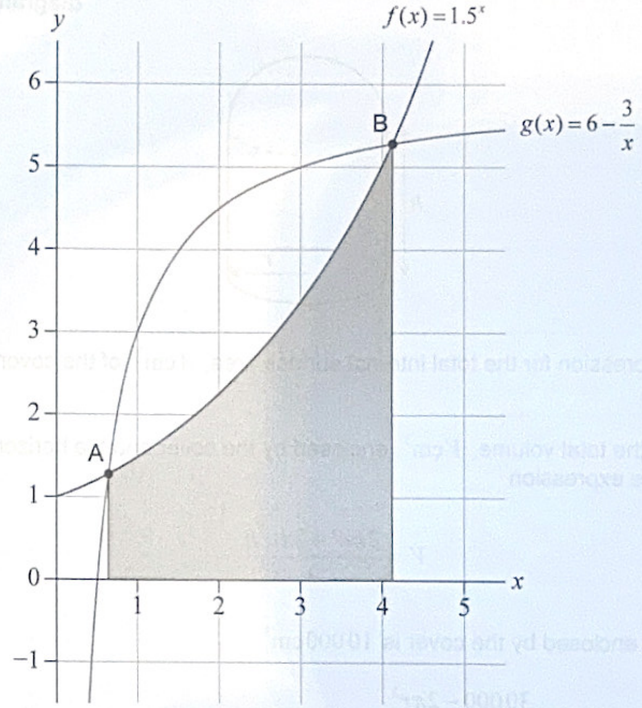
2. [Maximum mark: 12]

The diagram shows part of the graphs of the functions

$$f(x) = 1.5^x \qquad x \geq 0$$

$$g(x) = 6 - \frac{3}{x} \qquad x > 0.$$



(a) Solve $f(x) = g(x)$. [3]

(b) (i) Write down the integral that represents the area of the shaded region.

(ii) Calculate the area of this shaded region.

(iii) Hence, or otherwise, calculate the area of the region enclosed between the curves $y = f(x)$ and $y = g(x)$. [6]

The tangent to the graph of $y = f(x)$ is parallel to the tangent to the graph of $y = g(x)$ at $x = k$.

(c) Find the value of k. [3]

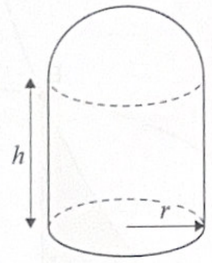
3. [Maximum mark: 16]

Ju Shen designs a plastic cover, in the shape of a cylinder combined with a hemisphere on top, as shown in the diagram.

The plastic used to make the cover forms the curved surface of both the hemisphere and the cylinder; there is no bottom to the cover, however it stands on a flat horizontal surface.

Let the height of the cylinder be h cm and the internal radius of its base be r cm.

diagram not to scale



(a) Find an expression for the total internal surface area, $A\,\text{cm}^2$, of the cover in terms of r and h. [2]

(b) Show that the total volume, $V\,\text{cm}^3$, enclosed by the cover and the horizontal surface is given by the expression

$$V = \frac{2\pi r^3 + 3\pi r^2 h}{3}.$$ [2]

The total volume enclosed by the cover is $10\,000\,\text{cm}^3$.

(c) Hence show that $h = \frac{30\,000 - 2\pi r^3}{3\pi r^2}$. [2]

Ju Shen uses the total internal surface area to model the amount of plastic used to construct the cover.

(d) Show that A is given by the expression

$$A = \frac{2\pi r^2}{3} + \frac{20\,000}{r}.$$ [2]

(This question continues on the following page)

(Question 3 continued)

Ju Shen wants to use the minimum amount of plastic in the construction of the cover.

(e) Find an expression for $\frac{dA}{dr}$. [3]

(f) Find the value of r and the value of h that minimizes the use of plastic. [4]

(g) By interpreting your answer to part (f), suggest the best shape for Ju Shen's plastic cover. [1]

4. [Maximum mark: 13]

A wind farm consists of five wind turbines, located at points A to E.

The table below shows the distances, in kilometres, between each pair of turbines.

	A	B	C	D	E
A		0.90	0.88	1.56	0.86
B	0.90		0.74	0.94	1.28
C	0.88	0.74		0.78	0.62
D	1.56	0.94	0.78		1.36
E	0.86	1.28	0.62	1.36	

The turbines must all be connected by cables. However, there does not need to be a direct connection between every pair.

(a) Use Prim's algorithm, starting with vertex A, to find the minimum total length of cable required to connect the turbines. Show the order in which you added the vertices. [4]

The supervisor of the wind farm has a monitoring cabin located at point F. The distances from F to each turbine are shown in the table.

Turbine	A	B	C	D	E
Distance from F (km)	0.96	1.82	1.57	2.24	1.14

The supervisor wants to visit every turbine exactly once for inspection, starting and finishing at the cabin, and using the route of shortest possible length.

(b) By deleting vertex F, find a lower bound for the length of the shortest route. [2]

(c) Use the nearest neighbour algorithm starting at F to find an upper bound for the length of the shortest route. [3]

(This question continues on the following page)

(Question 4 continued)

The table below shows the lower bounds found by deleting each of the other five vertices, and the upper bounds found by starting at each of the other five vertices.

Vertex	A	B	C	D	E
Lower bound	5.02	4.86	5.02	4.90	4.84
Upper bound	6.36	6.36	7.13	7.22	6.82

The supervisor travels between the turbines at a constant speed of 28 km/h and spends 12 minutes inspecting each turbine.

(d) Based on all the information above, find the best possible upper and lower bounds for the shortest amount of time, T hours, required for the inspection. Write your answer as an inequality. [4]

5. [Maximum mark: 15]

A zoologist collects a sample of cane beetles. He measures their length and categorizes them as "small" meaning from 10 to 12 mm long, "medium" meaning from 12 to 16 mm long and "large" meaning from 16 to 18 mm long. He also notes their sex and records the frequencies in the following table.

		Length, x mm		
		Small $10 < x \leq 12$	Medium $12 < x \leq 16$	Large $16 < x \leq 18$
Sex	Female	42	25	19
	Male	61	27	12

(a) Find how many cane beetles are in the zoologist's sample. [1]

(b) Based on this data set, estimate the mean length of a cane beetle. [2]

Two female beetles are chosen at random with replacement from the sample.

(c) Find the probability that they are both categorized as small. [3]

(d) Test, at the 5% significance level, the null hypothesis that length category and sex are independent. State the p-value of your test and write your conclusion in context. Justify your answer. [4]

Let ϕ be the population proportion of cane beetles that are male.

(e) Test, at the 5% significance level, the hypothesis that more than 45% of cane beetles are male. Write the null and alternative hypotheses. State the p-value of your test and write your conclusion in context. [5]

6. [Maximum mark: 21]

A financial analyst models the change in value of one share, x dollars at time t minutes, after a report is released. She uses the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 0.$$

This equation can be written as the coupled differential equations

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -3x - 4y.$$

(a) Find the general solution for x. [5]

Initially $x = 0$ and $\frac{dx}{dt} = -1$.

(b) (i) Find an expression for x in terms of t.

(ii) Sketch x against t in the interval $0 \leq t \leq 4$. [6]

Once the report has been released, the analyst is going to buy some shares and then sell them later.

(c) (i) Use your graph to find how long after the report is released the analyst should wait to buy the shares in order to maximize her profit.

(ii) Find the upper limit of the profit the analyst can make per share. [4]

An improved model is written as

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = x\sin t.$$

The same initial conditions as above apply.

(d) Use Euler's method with a t-interval of 0.1 to predict the value of x when $t = 1$. [6]

7. [Maximum mark: 17]

On any given day, the probability that Emlyn charges his phone depends only on whether he charged it the previous day.

If he charged his phone the previous day, the probability he charges it today is 0.4.

If he did not charge his phone the previous day, the probability he charges it today is p.

On day n this can be represented using the vector $\boldsymbol{v}_n$ where

$$\boldsymbol{v}_n = \begin{pmatrix} \text{probability that Emlyn charges his phone on day } n \\ \text{probability that Emlyn does not charge his phone on day } n \end{pmatrix}$$

A Markov chain model is formed where

$$\boldsymbol{v}_{n+1} = \boldsymbol{M}\boldsymbol{v}_n.$$

Matrix $\boldsymbol{M}$ is of the form $\begin{pmatrix} a & p \\ b & 1-p \end{pmatrix}$.

(a) Write down the value of

(i) a.

(ii) b. [2]

(b) On day zero Emlyn charges his phone. Find the probability

(i) that Emlyn charges his phone on all days from $n=1$ to $n=4$.

(ii) that Emlyn charges his phone on day 4, when $p=0.7$. [5]

(c) Demonstrate that, for all values of p, one eigenvector of $\boldsymbol{M}$ is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and hence state the associated eigenvalue. [4]

(d) Find, in terms of p, the steady state probability that Emlyn charges his phone on a given day. [4]

In the long term, Emlyn wants to charge his phone on at least 60% of days.

(e) Find the minimum value of p required for this to occur. [2]

References:

1. naisupakit, 2016. *Triangle Pillow tradition native Thai style pillow*. [image online] Available at: https://www.gettyimages.co.uk/detail/photo/triangle-pillow-tradition-native-thai-style-pillow-royalty-free-image/623127206 [Accessed 9 April 2024]. SOURCE ADAPTED.